Controlled and Secure Direct Communication Using GHZ State and Teleportation

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A theoretical scheme for controlled and secure direct communication is proposed. The communication is based on GHZ state and controlled quantum teleportation. After insuring the security of the quantum channel (a set of qubits in the GHZ state), Alice encodes the secret message directly on a sequence of particle states in the GHZ state and transmits them to Bob, supervised by Charlie using controlled quantum teleportation. Bob can read out the encoded messages directly by the measurement on his qubits. In this scheme, the controlled quantum teleportation transmits Alice's message without revealing any information to a potential eavesdropper. Because there is not a transmission of the qubit carrying the secret messages between Alice and Bob in the public channel, it is completely secure for controlled and direct secret communication if a perfect quantum channel is used. The feature of this scheme is that the communication between two sides depends on the agreement of a third side.

Key words: Secure Direct Communication; GHZ State; Controlled Quantum Teleportation.

Cryptography is an art to ensure that the secret message is intelligible only for the two authorized parties of communication and can not be altered during the transmission. It is generally believed that cryptography schemes are only completely secure when the two communicating parties establish a shared secret key before the transmission of a message. It is trusted that the only proven secure crypto-system is the one-timepad scheme in which the secret key is as long as the message. But it is difficult to distribute securely the secret key through a classical channel. Fortunately, people did discover protocols for secure key distribution. As shown in a seminal paper by Bennett and Brassard in 1984 [1], Alice and Bob can establish a shared secret key by exchanging single qubits, physically realized by the polarization of photons, for example. The security of this quantum key distribution is guaranteed by the principle of quantum mechanics. Up to now there have been a lot of theoretical quantum key distribution schemes, such as in [1-19].

Recently, a novel quantum direct communication protocol has be presented [20-22] that allows secure direct communication, where there is no need for establishing a shared secret key and the message is deter-

ministically sent through the quantum channel, but can only be decoded after a final transmission of classical information. Boström and Felbinger [23] put forward a direct communication scheme, the "ping-pong protocol", which also allows for deterministic communication. This protocol can be used for the transmission of either a secret key or a plaintext message. In the latter case, the protocol is quasi-secure, i. e. an eavesdropper is able to gain a small amount of message information before being detected. In case of a key transmission the protocol is asymptotically secure if a perfect quantum channel is used. But it is insecure if it is operated in a noisy quantum channel, as indicated by Wójcik [24]. There is some probability that a part of the messages might be leaked to the eavesdropper, Eve, especially in a noisy quantum channel, because Eve can use the intercept-resending strategy to steal some secret messages even though Alice and Bob will find out her in the end of the communication. More recently Deng et al. [25] suggested a two-step quantum direct communication protocol using an Einstein-Podolsky-Rosen pair block. It was shown that it is provably secure. However in all these secure direct communication schemes it is necessary to send the qubits carrying secret messages

in the public channel. Therefore, Eve can attack the qubits in transmission. Yan and Zhang [26] presented a scheme for secure direct and confidential communication between Alice and Bob, using Einstein-Podolsky-Rosen pairs and teleportation [27]. Because there is not a transmission of the qubits carrying the secret messages between Alice and Bob in the public channel, it is completely secure for direct secret communication if a perfect quantum channel is used.

Quantum teleportation was invented by Bennett et al. [27] and developed by many authors [28–29]. The controlled quantum teleportation scheme was first presented by Karlsson and Bourennane [28], with very similar ideas also in the quantum secret sharing paper of Hillery et al. [19]. In [28, 19] the entanglement property of the GHZ state is utilized. According to the scheme, a third side is included, so that the quantum channel is supervised by this additional side. The signal state can not be transmitted unless all three sides agree to cooperate.

In this paper we design a scheme for controlled and secure direct communication based on the GHZ state and controlled quantum teleportation. The feature of this scheme is that the communication between two sides depends on the agreement of the third side.

The new protocol can be divided into two steps, one is to prepare a set of triplets of qubits in the GHZ state (quantum channel), the other is to transmit messages using controlled quantum teleportation.

Preparing of the Quantum Channel

Suppose that Alice, Bob, and Charlie share a set of triplets of qubits in the GHZ state

$$|\Phi^{+}\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{ABC}.$$
 (1)

Obtaining these triplets of particles in the GHZ state could have come about in many different ways; for example, Alice, Bob or Charlie could prepare the triplets and then send one qubit of each triplet to each of the other two persons (that is, one of them generates and shares each of the triplets with the other two people.). Alternatively, a fourth party could prepare an ensemble of particles in the GHZ state, and ask Alice, Bob, and Charlie to each take a particle (A, B, C, respectively) in each triplet. Or they could have met a long time ago and shared them, storing them until the present. Alice, Bob, and Charlie then choose randomly a subset of qubits in the GHZ state, and do some appropri-

ate tests of fidelity. Passing the test certifies that they continue to hold sufficiently pure, entangled quantum states. However, if tampering has occurred, Alice, Bob, and Charlie discard these triplets, and a new set of qubits in the GHZ state should be constructed.

Secure Direct Communication Using Controlled Teleportation

After insuring the security of the quantum channel (GHZ state), we begin controlled and secure direct communication. Suppose that Alice has a particle sequence and she wishes to communicate information to Bob supervised by Charlie. First Alice makes her particle sequence in the states, composed of $|+\rangle$ and $|-\rangle$, according to the message sequence. For example, if the message to be transmitted is 101001, then the sequence of particle states should be in the state $|+\rangle|-\rangle|+\rangle|-\rangle|+\rangle$, i.e. $|+\rangle$ and $|-\rangle$ correspond to 1 and 0 respectively. Here

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \quad (2)$$

Remarkably, quantum entanglement of the GHZ state can serve as a channel for transmission of a message encoded in the sequence of particle states. This is the process called controlled quantum teleportation [28, 19], which we now describe. Suppose the quantum channel $|\Phi^+\rangle_{ABC}$ shared by particles A, B and C belong to Alice, Bob, and Charlie, respectively. In components we write the signal state carrying a secret message

$$|\Psi\rangle_{\rm D} = \frac{1}{\sqrt{2}}(|0\rangle + b|1\rangle)_{\rm D},$$
 (3)

where b=1 and b=-1 correspond to $|+\rangle$ and $|-\rangle$ respectively. The quantum state of the whole system (the four qubits) can be written as

$$\begin{split} |\Psi\rangle_{\rm D}|\Phi^{+}\rangle_{\rm ABC} \\ &= \frac{1}{\sqrt{2}}(|0\rangle + b|1\rangle)_{\rm D}\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{\rm ABC} \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{\rm DA}\frac{1}{\sqrt{2}}(|00\rangle - b|11\rangle)_{\rm BC} \\ &+ \frac{1}{2} \cdot \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{\rm DA}\frac{1}{\sqrt{2}}(|00\rangle + b|11\rangle)_{\rm BC} \\ &+ \frac{1}{2} \cdot \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{\rm DA}\frac{1}{\sqrt{2}}(b|00\rangle - |11\rangle)_{\rm BC} \\ &+ \frac{1}{2} \cdot \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{\rm DA}\frac{1}{\sqrt{2}}(-b|00\rangle - |11\rangle)_{\rm BC}. \end{split}$$

Now Alice performs a Bell state measurement [30,31] on the qubits DA and then broadcasts the outcome of her measurement. Depending on Alice's four possible measurement outcomes $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{DA},\,\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)_{DA},\,\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)_{DA}$ and $\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)_{DA},$ Bob and/or Charlie can transform the qubits BC to the common form

$$|\Psi\rangle_{\rm BC} = \frac{1}{\sqrt{2}}(|00\rangle + b|11\rangle)_{\rm BC} \tag{5}$$

by the corresponding transformations $I_B \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_C$, $I_B \otimes I_C$, $(|0\rangle\langle 1| + |1\rangle\langle 0|)_B \otimes (-|0\rangle\langle 1| + |1\rangle\langle 0|)_C$ and $(-|0\rangle\langle 1| - |1\rangle\langle 0|)_B \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)_C$, respectively.

As a matter of fact, at this moment, neither Bob nor Charlie can obtain the signal state $\frac{1}{\sqrt{2}}(|0\rangle+b|1\rangle)$ without the cooperation of the other one.

If Charlie would like to help Bob for the quantum teleportation, he should just measure his portion of BC, namely qubit C, on the base $\{|+\rangle_C, |-\rangle_C\}$, and transfer the result of his measurement to Bob via a classical channel. Here the state of the qubits BC can be expressed as:

$$|\Psi\rangle_{BC} = \frac{1}{\sqrt{2}}(|00\rangle + b|11\rangle)_{BC}$$

$$= \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}}(|0\rangle + b|1\rangle)_{B} |+\rangle_{C} + \frac{1}{\sqrt{2}}(|0\rangle - b|1\rangle)_{B} |-\rangle_{C} \right]. \tag{6}$$

Once Bob has learned Charlie's result, he can "fix up" his state, recovering $|\Psi\rangle_D$, by applying an appropriate unitary transformation. In fact, according to the two possible results $|+\rangle_C$ and $|-\rangle_C$, Bob can perform the corresponding transformations I_B and $(|0\rangle\langle 0|-|1\rangle\langle 1|)_B$, respectively, on qubit B to obtain the signal state,

$$|\Psi\rangle_{\rm B} = \frac{1}{\sqrt{2}}(|0\rangle + b|1\rangle)_{\rm B}.$$
 (7)

Then Bob measures the basis $\{|+\rangle, |-\rangle\}$ and reads out the messages that Alice wants to transmit to him.

It is undeniable that this process of controlled quantum teleportation has similar notable features of the original quantum teleportation [27] which was mentioned in [26]. For instance, the process is entirely unaffected by any noise in the spatial environment

between each other, and the controlled teleportation achieves perfect transmission of delicate information across a noisy environment and without even knowing the locations of each other. In the process Bob is left with a perfect instance of $|\Psi\rangle$ and hence no participants can gain any further information about its identity. So in our scheme controlled quantum teleportation transmits Alice's message without revealing any information to a potential eavesdropper, Eve, if the quantum channel is in perfect GHZ state (perfect quantum channel).

The security of this protocol only depends on the perfect quantum channel (pure GHZ state). Thus, as long as the quantum channel is perfect, our scheme is absolutely reliable, deterministic and secure.

Of course we should pointed out that testing of the security of the quantum channel is necessary, since a potential eavesdropper may obtain information as following:

(1) Eve can use the entanglement triplet in the GHZ state to obtain information. Suppose that Eve has triplets of qubits EFG in the state $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{EFG}$. When Eve obtains particles B and C (the other cases are similar to this) in preparing the GHZ state, she performs a measurement on the particles BCE using the base $\{\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle), \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \frac{1}{\sqrt{2}}(|001\rangle - |110\rangle), \frac{1}{\sqrt{2}}(|001\rangle + |110\rangle), \frac{1}{\sqrt{2}}(|100\rangle - |011\rangle), \frac{1}{\sqrt{2}}(|100\rangle + |101\rangle)$. From the expression

$$\begin{split} |\Phi^{+}\rangle_{ABC}|\Phi^{+}\rangle_{EFG} &= \\ \frac{1}{2} \Big[\frac{1}{\sqrt{2}} (|000\rangle - |111\rangle)_{BCE} \frac{1}{\sqrt{2}} (|000\rangle - |111\rangle)_{AFG} \\ &+ \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{BCE} \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)_{AFG} \\ &+ \frac{1}{\sqrt{2}} (|001\rangle - |110\rangle)_{BCE} \frac{1}{\sqrt{2}} (-|011\rangle + |100\rangle)_{AFG} \\ &+ \frac{1}{\sqrt{2}} (|001\rangle + |110\rangle)_{BCE} \frac{1}{\sqrt{2}} (-|011\rangle - |100\rangle)_{AFG} \Big]. \end{split}$$

we can read off the possible post-measurement states of the particles AFG $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{AFG}$, $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{AFG}$, $\frac{1}{\sqrt{2}}(-|011\rangle + |100\rangle)_{AFG}$, depending on Eve's possible measurement outcomes $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)_{BCE}$,

 $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{BCE},~\frac{1}{\sqrt{2}}(|001\rangle-|110\rangle)_{BCE}$ and $\frac{1}{\sqrt{2}}(|001\rangle+|110\rangle)_{BCE}$, respectively. Then Eve transmits the particles B and C to Bob and Charlie, respectively. Alice, Bob, and Charlie proceed as usual, since they do not know that there is a potential eavesdropper intercepting and resending their particles if they do not test the quantum channel. Therefore a part of messages might be leaked to Eve.

However, by testing the quantum channel, Alice, Bob, and Charlie can find Eve and avoid the information being leaked. In fact after the measurement performed by Eve, there is not any correlation between the particles A and B and particles A and C. So, when Alice, Bob, and Charlie perform measurements on oneself's particle using the base $\{|0\rangle, |1\rangle\}$ independently, the results will be random without any correlation. If this case occurred, they can assert that an eavesdropper exists, and the triplets in the GHZ state should be discarded.

(2) Eve can obtain information by coupling the qubits in the GHZ state with her probe in preparing the GHZ state. In this case Alice, Bob, and Charlie can also test whether the quantum channel is perfect or not by the following strategy: They select at random a subset of triplets of qubits in the GHZ state. All three measure σ_x on some of the particles at their disposal, σ_y on the others, and then inform each other of the measurement outcomes and the corresponding operators. When two of the friends measure σ_v and the third measures σ_x on a triplet, and all three of them measure σ_x on triplet, it just so happens that $|\Phi^+\rangle_{ABC}$ is an eigenstate of the three operator products $\sigma_y^A\sigma_x^B\sigma_y^C$, $\sigma_y^A\sigma_y^B\sigma_x^C$, $\sigma_x^A\sigma_y^B\sigma_y^C$ with eigenvalue 1 and is also an eigenstate of $\sigma_r^A \sigma_r^B \sigma_r^C$ with eigenvalue -1. (Here σ_x^A Alice's spin, σ_x^B operates on Bob's, etc.) Thus, if Alice, Bob, and Charlie all measure σ_x , they may obtain -1, -1, -1 or -1, 1or 1, -1, 1 or 1, 1, -1 respectively; if two measure σ_v and the third measures σ_x , they may obtain 1, 1, 1 or 1, -1, -1 or -1, 1, -1 or -1, -1, 1 respectively; only these results are possible (i.e. these results are complete correlation.). Here we can say that Eve does not exist. However, if other outcomes appear or measurement outcomes are not complete correlation, they can affirm that a potential Eve exists and has coupled the triplets of qubits in the GHZ state with her probe. The reason is as follows:

As a matter of fact, the overall state of the qubits of Alice, Bob, Charlie, and Eve in general form is

$$|\Psi\rangle_{ABCE} = |000\rangle|e_{000}\rangle + |001\rangle|e_{001}\rangle + |010\rangle|e_{010}\rangle + |011\rangle|e_{011}\rangle + |100\rangle|e_{100}\rangle + |101\rangle|e_{101}\rangle + |110\rangle|e_{110}\rangle + |111\rangle|e_{111}\rangle,$$
(9)

where $|e_{ijk}\rangle$ (i, j, k = 0, 1) is a state of Eve's particles. Suppose that $|\Psi\rangle_{ABCE}$ is an eigenstate of $\sigma_x^A \sigma_x^B \sigma_x^C$ with eigenvalue -1, it must be

$$\begin{split} \Psi\rangle_{\text{ABCE}} &= \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)|e'_{000}\rangle \\ &+ \frac{1}{\sqrt{2}}(|001\rangle - |110\rangle)|e'_{001}\rangle \\ &+ \frac{1}{\sqrt{2}}(|010\rangle - |101\rangle)|e'_{010}\rangle \\ &+ \frac{1}{\sqrt{2}}(|011\rangle - |100\rangle)|e'_{011}\rangle. \end{split} \tag{10}$$

At the same time, assume that $|\Psi\rangle_{ABCE}$ is also an eigenstate of $\sigma_y^A\sigma_x^B\sigma_y^C$, $\sigma_y^A\sigma_y^B\sigma_x^C$, $\sigma_x^A\sigma_y^B\sigma_y^C$ with eigenvalue 1, so $|\Psi\rangle_{ABCE}$ must be

$$|\Psi\rangle_{\text{ABCE}} = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)|e_{000}''\rangle.$$
 (11)

From this fact we conclude that as long as $|\Psi\rangle_{ABCE}$ is the simultaneous eigenstate of the operators $\sigma_y^A\sigma_x^B\sigma_y^C$, $\sigma_y^A\sigma_y^B\sigma_x^C$, $\sigma_x^A\sigma_y^B\sigma_y^C$ and $\sigma_x^A\sigma_x^B\sigma_x^C$ with the eigenvalues 1, 1, 1, and -1 respectively, there is no entanglement between Alice, Bob, and Charlie's particles and Eve's particles. So, when Alice, Bob, and Charlie confirm that their qubits are in complete correlation, then Eve can not obtain any information. If the situation is not the case, evidently there is a potential eavesdropper. We should abandon the quantum channel.

In one word, under any case, as long as an eavesdropper exists, we can find her and insure the security of the quantum channel to realize controlled and secure direct communication.

In summary, we give a scheme for controlled and secure direct communication. The communication is based on the GHZ state and controlled teleportation. After insuring the security of the quantum channel (GHZ states), Alice encodes the secret messages directly on a sequence of particle states in the GHZ state and transmits them to Bob by teleportation supervised by Charlie. Evidently controlled teleportation transmits Alice's messages without revealing any information to a potential eavesdropper. Bob can read out the

encoded messages directly by the measurement on his qubits. Because there is not a transmission of the qubit which carries the secret message between Alice and Bob, it is completely secure for controlled and direct secret communication if a perfect quantum channel is used.

Teleportation has been realized in the experiments [32-34], therefore we hope that our protocol for con-

trolled and secure direct communication will be realized by experiment in the near future.

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- C. H. Bennett and G. Brassard, Proc. IEEE Int. Conf. on Computers, Systems and Signal Processing, IEEE, New York 1984, p. 175.
- [2] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
- [3] C. H. Bennett, G. Brassard, and N. D. Mermin, Phys. Rev. Lett. 68, 557 (1992).
- [4] C. H. Bennett, Phys. Rev. Lett. 68, 3121 (1992).
- [5] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
- [6] L. Goldenberg and L. Vaidman, Phys. Rev. Lett. 75, 1239 (1995).
- [7] B. Huttner, N. Imoto, N. Gisin, and T. Mor, Phys. Rev. A 51, 1863 (1995).
- [8] M. Koashi and N. Imoto, Phys. Rev. Lett. 79, 2383 (1997).
- [9] D. Bruß, Phys. Rev. Lett. 81, 3018 (1998).
- [10] W. Y. Hwang, I. G. Koh, and Y. D. Han, Phys. Lett. A 244, 489 (1998).
- [11] A. Cabello, Phys. Rev. Lett. 85, 5635 (2000).
- [12] A. Cabello, Phys. Rev. A 61, 052312 (2000).
- [13] G.L. Long and X.S. Liu, Phys. Rev. A **65**, 032302 (2002)
- [14] B. S. Shi, Y. K. Jiang, and G. C. Guo, Appl. Phys. B 70, 415 (2000).
- [15] P. Xue, C.F. Li, and G.C. Guo, Phys. Rev. A 65, 022317 (2002).
- [16] F. G. Deng, et al. Chin. Phys. Lett. 19, 893 (2002).

- [17] S. J. D. Phoenix, S. M. Barnett, P. D. Townsend, and K. J. Blow, J. Modern Optics 42, 1155 (1995).
- [18] H.-K. Lo, H. F. Chan, and M. Ardehali, arXiv quantph/0011056.
- [19] M. Hillery, V. Bužek, and A. Berthiaume, Phys. Rev. A 59, 1829 (1999).
- [20] K. Shimizu and N. Imoto, Phys. Rev. A 60, 157 (1999).
- [21] K. Shimizu and N. Imoto, Phys. Rev. A 62, 054303 (2000).
- [22] A. Beige, et al. Acta Phys. Pol. A 101, 357 (2002).
- [23] K. Boström and T. Felbinger, Phys. Rev. Lett. 89, 187902 (2002).
- [24] A. Wójcik, Phys. Rev. Lett. 90, 157901 (2003).
- [25] F. G. Deng, G. L. Long, and X. S. Liu, Phys. Rev. A 68, 042317 (2003).
- [26] F. L. Yan and X. Q. Zhang, arXiv: quant-ph/0311132.
- [27] C. H. Bennett, et al. Phys. Rev. Lett. 70, 1895 (1993).
- [28] A. Karlsson and M. Bourennane, Phys. Rev. A 58, 4394 (1998).
- [29] F. L. Yan and D. Wang, Phys. Lett. A 316, 297 (2003).
- [30] H. Weinfurter, Europhys. Lett. 25, (1994) 559.
- [31] J. W. Pan and A. Zeilinger, Phys. Rev. A 57, 2208 (1998).
- [32] D. Bouwmeester, et al., Nature, London 390, 575 (1997).
- [33] D. Boschi, et al., Phys. Rev. Lett. 80, 1121 (1998).
- [34] M. A. Nielsen, et al., Nature, London **396**, 52 (1998).